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Rotating Solutions of Einstein-Maxwell-Dilaton Gravity with Unusual Asymptotics

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We study electrically charged, dilaton black holes, which possess infinitesimal angular momentum in the presence of one or two Liouville type potentials. These solutions are neither asymptotically flat nor (anti)-de Sitter. Some properties of the solutions are discussed.

KEY WORDS: black holes; Dilaton gravity; AdS/CFT correspondence.

1. INTRODUCTION

Recently, non-asymptotically flat black hole spacetimes have been of much interest in the framework of AdS/CFT correspondence. Black hole spacetimes which are neither asymptotically flat nor dS/AdS have been found and investigated by many authors. The first uncharged solutions were found by Mignemi and Wiltshire (1989), Wiltshire (1991) and Mignemi and Wiltshire (1992). Global properties of the Einstein-Maxwell-dilaton (EMd) gravity with a Liouville potential were first obtained and discussed by Poletti and Wiltshire (1994, 1995). Static, electrically or magnetically charged, non-asymptotically flat, non-dS/AdS black holes in various dimensions were also found and discussed in (Cai and Wang, 2004). The exact static and spherically symmetric solutions of the electrically or magnetically charged dilaton black holes in n dimensions in the presence of one and two Liouville type potentials and with unusual asymptotics (neither flat nor (anti) de Sitter) were introduced by Chan *et al.* (1995).

The exact solutions mentioned above are all static. Recently, magnetic, rotating solutions in four dimensional Einstein-Maxwell-dilaton gravity with Liouville-type potential has been constructed by Dehghani (2005). These solutions are not black holes, and present spacetimes with conic singularity. Electrically charged

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rotating dilaton black strings were also obtained and discussed in (Dehghani and Farhangkhah, 2005). Till now, charged rotating dilaton black hole solutions for an arbitrary coupling constant have not been constructed. Indeed, exact solution for a rotating black hole with a special dilaton coupling was derived using the inverse scattering method (Belinsky and Ruffini, 1980). For general dilaton coupling, the properties of asymptotically flat charged dilaton black holes only with infinitesimal angular momentum (Horne and Horowitz, 1992; Shiraishi,1992) or small charge Casadio *et al.* (1997) have been investigated. Stationary rotating black holes in $SU(2)$ Einstein-Yang-Mills theory, coupled to a dilaton considered by Kleihaus *et al.* (2004). These black holes possess non-trivial non-Abelian electric and magnetic fields outside their regular event horizon. Some classes of solutions of non asymptotically flat, non AdS/dS charged dilaton black holes with infinitesimal small angular momentum, and with one Liouville type potential, were also discussed by Ghosh and Mitra (2003). In this paper, we present electrically charged, dilaton black holes, for an arbitrary value of coupling constant with infinitesimal angular momentum. We consider three cases: without potential, with one Liouville type potential and two Liouville type potential. These solutions are neither asymptotically flat nor AdS/dS.

The organization of this paper is as follows: After introducing the general equations of motion, we present and discuss rotating dilaton black holes without potential. In section 4, we present two classes of rotating solutions with a Liouville type potential and general dilaton coupling. In section 5, we generalized these rotating solutions for the case of two Liouville potentials. The last section is devoted to some concluding remarks.

2. FIELD EQUATIONS

We consider the four-dimensional action in which gravity is coupled to dilaton and Maxwell fields with an action

$$S = \int d^4x\sqrt{-g}\left[\mathcal{R} - 2(\nabla\phi)^2 - V(\phi) - e^{-2\alpha\phi}F^{\mu\nu}F_{\mu\nu}\right] \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar curvature and ϕ is the dilaton field and $V(\phi)$ is a potential for ϕ. The equations of motion can be obtained by varying the action (1) with respect to the gauge field A_μ, the metric $g_{\mu\nu}$ and the dilaton field ϕ which yields the following field equations

$$\mathcal{R}_{\mu\nu} = 2\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g_{\mu\nu}V(\phi) + 2e^{-2\alpha\phi}\left(F_{\mu\eta}F_\nu^\eta - \frac{1}{4}g_{\mu\nu}F_{\lambda\eta}F^{\lambda\eta}\right), \tag{2}$$

$$\partial_\mu(\sqrt{-g}e^{-2\alpha\phi}F^{\mu\nu}) = 0, \tag{3}$$

$$\nabla^\mu\nabla_\mu\phi = \frac{1}{4}\frac{\partial V}{\partial\phi} - \frac{\alpha}{2}e^{-2\alpha\phi}F_{\lambda\eta}F^{\lambda\eta}. \tag{4}$$

We wish to find rotating solutions of the above field equations. For infinitesimal angular momentum, we can take the metric of the form

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) - 2af(r)\sin^2\theta\, dt\, d\varphi. \tag{5}$$

Here, a is an angular momentum parameter and $f(r)$ is a function to be determined. Note that, in the particular case $a = 0$, this metric reduces to the static and spherically symmetric case. For small a, we except to have solutions with $U(r)$ still a function of r alone.

First of all, the t component of the Maxwell equation can be integrated immediately to give

$$F^{rt} = \frac{qe^{2\alpha\phi}}{R^2}, \tag{6}$$

where q, is the electric charge. In general, in the presence of rotation, there is a vector potential

$$A_\varphi = aqh(r)\sin^2\theta. \tag{7}$$

With the metric (5) and Maxwell fields (6) and (7), the field equations reduce to the following system of coupled ordinary differential equations

$$\frac{1}{R^2}\frac{d}{dr}\left(R^2 U\frac{d\phi}{dr}\right) = \frac{1}{4}\frac{dV}{d\phi} + \alpha e^{2\alpha\phi}\frac{q^2}{R^4}, \tag{8}$$

$$\frac{1}{R}\frac{d^2R}{dr^2} + \left(\frac{d\phi}{dr}\right)^2 = 0, \tag{9}$$

$$\frac{1}{R^2}\frac{d}{dr}\left(U\frac{dR^2}{dr}\right) = \frac{2}{R^2} - V(\phi) - 2e^{2\alpha\phi}\frac{q^2}{R^4} \tag{10}$$

$$R^4\frac{d^2U}{dr^2} - 2R^2U\left(\frac{dR}{dr}\right)^2 - 2R^3U\left(\frac{d^2R}{dr^2}\right) + 2R^2 - 4q^2e^{2\alpha\phi} = 0, \tag{11}$$

In addition, we have two coupled differential equations for arbitrary functions $f(r)$ and $h(r)$.

$$R^2\frac{d^2f}{dr^2} - 2f\left(\frac{dR}{dr}\right)^2 - 2fR\frac{d^2R}{dr^2} - 4q^2\frac{dh}{dr} = 0, \tag{12}$$

$$R^2\frac{d}{dr}\left(Ue^{-2\alpha\phi}\frac{dh}{dr}\right) - R^2\frac{d}{dr}\left(\frac{f}{R^2}\right) - 2he^{-2\alpha\phi} = 0, \tag{13}$$

These equations which arise from the presence of A_φ, appear only when $a \neq 0$, while the other equations were there also in the static, spherically symmetric case. Static solutions of these equations with unusual asymptotic were given in Chan *et al.* (1995).

To solve these equations, we make the ansatz

$$R(r) = e^{\alpha\phi(r)}, \tag{14}$$

Using (14) in equation (9), immediately gives

$$\phi(r) = \frac{\alpha}{1+\alpha^2} \ln(br - c), \tag{15}$$

where b and c are integration constants. For later convenience, without loss of generality, we set $b = 1$ and $c = 0$.

3. SOLUTIONS WITH $V(\phi) = 0$

We begin by looking for the solutions without Liouville potential ($V(\phi) = 0$). In this case Equations (8)–(11), gives the following solution

$$U(r) = r^{2-2N}\left(1 - \frac{2M}{Nr}\right), \tag{16}$$

with $N = \frac{\alpha^2}{1+\alpha^2}$ and M is the quasilocal mass (Brown and York, 1993). Note that the solution is ill defined for $\alpha = 0$. In order, that this solution satisfy in all field equations, we should have the following relation for the electric charge:

$$q^2 = \frac{1}{1+\alpha^2}, \tag{17}$$

There is an event horizon at $r_+ = \frac{2M}{N}$. In the limit $\alpha^2 \to \infty$, the electric charge vanishes and the metric reduces to the Schwarzschild black hole.

Here we are interested in finding rotating version of these solutions. For infinitesimal rotation parameter a, we can get a solution of the full set of equations with $U(r)$, $R(r)$ and $\phi(r)$ unaltered and supplemented by the solutions of the new Equations (12) and (13) for two unknown functions $f(r)$ and $h(r)$. To solve these equations we try the power relation

$$h(r) = kr^m, \tag{18}$$

where k is a constant. Using this anzats, we can distinguish different solutions:

For $m = 0$, one obtains the following solutions for $f(r)$:

$$f_1(r) = \frac{2k}{4N-1} r^{1-2N}, \tag{19}$$

$$f_2(r) = cr^{2N} + f_1(r). \tag{20}$$

where c is an integration constant. Note that these solutions are ill defined for $\alpha^2 = \frac{1}{3}$ and in the case $\alpha = 1$, $f_1(r)$ becomes constant, while $f_2(r) = cr$.

In the particular case $k = 0$, this solution involves a change of the metric from the non-rotating form without any change of the Maxwell field and follows from the general structure of the new equations for $f(r)$ and $h(r)$. This may be surprising at first sight because a rotation enters the metric without any rotation in the charge; this is possible because the function $f(r)$ does not obey conventional boundary conditions for large r and in fact increases with r.

In the large α limit, from (17) we have $q^2 = 0$. For $f(r)$, one obtains

$$f(r) = -2kmMr^m + cr^2, \tag{21}$$

with c constant and $m = -1, 2$. This is similar to the slowly rotating Kerr black hole. The corresponding static case is the Schwarzschild black hole.

In addition, there are *asymptotic* solutions such as

$$f(r) = r^{2N}\left(c + \frac{4kq^2}{\gamma}r^\gamma\right), \tag{22}$$

with $\gamma = m + 1 - 4N$ and c is a constant. In this case, α is related to m via

$$\alpha^2 = \frac{m^2 + m - 6}{3m - m^2 + 2}. \tag{23}$$

Note that this solution is ill defined for $\gamma = 0$ or $\alpha^2 = \frac{m+1}{3-m}$.

4. SOLUTION WITH A LIOUVILLE TYPE POTENTIAL

In this section, we consider the action (1) with a Liouville type potential,

$$V(\phi) = 2\Lambda e^{2\beta\phi}, \tag{24}$$

where Λ and β are constants. In this case, Equations (8)–(11) admit two classes of solutions.

i) For the first class of solutions, we obtain

$$U(r) = r^{2-2N}\left(1 - \frac{2M}{Nr} + \frac{\Lambda(1+\alpha^2)^2}{\alpha^2(1-3\alpha^2)}r^{2(2N-1)}\right), \tag{25}$$

with $\beta = \frac{-1}{\alpha}$. In order, to satisfy this solution in all field equations, the electric charge should be related to α via Equation (17). Note that the solution is ill defined for $\alpha^2 = \frac{1}{3}$ and $\alpha = 0$. In the limit $\Lambda \to 0$ the solution reduces to that with $V(\phi) = 0$. On the other hand, when $\alpha^2 \to \infty$ the solution becomes

$$U(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2, \tag{26}$$

which is the Schwarzschild dS/AdS black hole, depending on the sign of Λ. In order to investigate the causal structure of the solution, we must investigate the zeros of the metric function $U(r)$. In fact, for $0 < r < \infty$ the zeros of $U(r)$ are governed by the function

$$F(r) = 1 - \frac{2M}{Nr} + \frac{\Lambda(1+\alpha^2)^2}{\alpha^2(1-3\alpha^2)} r^{2(2N-1)}. \tag{27}$$

We investigate the function $g(r) = rF(r)$, for simplicity. The cases with $\alpha^2 > 1/3$ and $\alpha^2 < 1/3$ should be considered separately. We should also consider the sign of the parameter Λ in each case. In the first case where $\alpha^2 < 1/3$ and $\Lambda < 0$ we may have one horizon since $\frac{dg}{dr} > 0$. But the more interesting case happens for $\Lambda > 0$ where we obtain only one local minimum at $r = r_{\min}$ where

$$r_{\min} = \left(\frac{\alpha^2}{\Lambda(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} \tag{28}$$

The function $g(r)$ possesses zeros if $g(r_{\min}) \leq 0$. There are two zeros for $g(r_{\min}) < 0$ and one degenerate zero for $g(r_{\min}) = 0$ which corresponds to an extremal black hole. The condition $g(r_{\min}) \leq 0$ gives

$$M \geq \frac{\alpha^2(2-3\alpha^2)}{2(1+\alpha^2)(1-3\alpha^2)} \left(\frac{\alpha^2}{\Lambda(1+\alpha^2)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} \tag{29}$$

In the second case where $\alpha^2 > 1/3$ and $\Lambda < 0$, the function $g(r)$ increases monotonically. So we can conclude that there is one point where $g(r) = 0$ which is the black hole horizon. For $\Lambda > 0$ we find local extremum for the function. The sign of $\frac{d^2 g(r)}{d^2 r}$ determines weather we have local maximum or minimum. For $\alpha^2 > 1$ we have local maximum and $g(r_{\max})$ should be positive in order to have any horizon. The latter condition gives

$$M \leq \frac{\alpha^2(2-3\alpha^2)}{2(1+\alpha^2)(1-3\alpha^2)} \left(\frac{\alpha^2}{\Lambda(1+\alpha^2)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} \tag{30}$$

If we have $\frac{1}{3} < \alpha^2 < 1$, then we would have local minimum and in case of any horizon existing $g(r_{\min})$ in Equation (28) should be negative which implies Equation (29). The above considerations show that the solutions describe black holes with two horizons or an extremal black hole hiding a singularity at the origin $r = 0$, when the mass satisfies (29) or (30). The radius of the inner and outer horizons can not be expressed in a closed analytical form except for the extremal case. The radius of the extremal solution coincides with $r_{\min}$

$$r_{\text{ext}} = r_{\min} = \left(\frac{\alpha^2}{\Lambda(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} = \frac{2(1+\alpha^2)(1-3\alpha^2)}{\alpha^2(2-3\alpha^2)} M \tag{31}$$

Unfortunately, because of the nature of the exponents of r in (27), the event horizon determined by $F(r) = 0$ can not be expressed in a closed analytical form for arbitrary α.

In order to obtain the rotating version of this solutions, we must solve Equations (12) and (13) for the two unknown functions $f(r)$ and $h(r)$. Using the anzats (18) we can distinguish different solutions:

For $m = 0$, once again we have solution of the form (19)

$$f_1(r) = \frac{2k}{4N-1} r^{1-2N}, \tag{32}$$

$$f_2(r) = cr^{2N} + f_1(r). \tag{33}$$

In particular case $k = 0$, we have unusual solution where there is a change in the metric from the non-rotating form without any change in the Maxwell field.

In the large α limit, we have a solution for $m = -1$. In this case, from (17) we have $q^2 = 0$. For $f(r)$, one obtains

$$f(r) = \frac{2kM}{r} + \frac{\Lambda k}{3} r^2 + cr^2, \tag{34}$$

with c constant. This is the form for slowly rotating Kerr Ads/ds black hole. The corresponding static case is the Schwarzschild AdS/dS black hole which present in (26).

In addition, for $\alpha^2 < 1$ there are *asymptotic* solutions such as

$$f(r) = r^{2N} \left(c + \frac{4kq^2}{\gamma} r^{\gamma} \right), \tag{35}$$

with $\gamma = m + 1 - 4N$ and c constant.

For $\alpha^2 = 1$, there exist an exact solutions such as

$$f(r) = \frac{4kq^2}{m-1} r^m - 4\,kmMr^{m-1} + cr, \tag{36}$$

with $\Lambda = \frac{m^2-m-4}{2m(m-1)}$. Note that for $m \neq 2$ this solution exists only for $M = 0$. For $m = 0, 1$, the solution doesn't exist since Λ diverges.

ii) For the second class of solutions, we obtain

$$U(r) = r^{2-2N} \left(1 - 2\Lambda - \frac{2M}{Nr} \right), \tag{37}$$

with $\beta = -\alpha$. For the electric charge one obtains

$$q^2 = \frac{1 + \Lambda(\alpha^2 - 1)}{1 + \alpha^2}. \tag{38}$$

There is an event horizon at $r_+ = \frac{2M}{N(1-2\Lambda)}$ which is regular only for $\Lambda < \frac{1}{2}$. In the limit $\Lambda \to 0$ this solution reduces to that with $V(\phi) = 0$. Here we are interested in finding the rotating version of this static solution, that is to say, in solving the corresponding coupled equations for two unknown functions $f(r)$ and $h(r)$. By the anzats (18) we can distinguish different solutions:

For $m = 0$, once again we have solution of the form (19). In the large α limit, and $q = 0$ we have also solutions as long as

$$f(r) = -2kmMr^m + cr^2, \tag{39}$$

with c constant and $\Lambda = \frac{m^2-m-4}{2m(m-1)}$. Note that for $m \neq 2, -1$ this solution exist only for $M = 0$. For $m = 0, 1$, the solution doesn't exist since Λ diverges. For *asymptotic* solutions once again we have solutions of the form

$$f(r) = r^{2N}\left(c + \frac{4kq^2}{\gamma}r^\gamma\right), \tag{40}$$

with $\gamma = m + 1 - 4N$ and c constant. In this case, Λ is related to m and α via the

$$\Lambda = \frac{(m^2+m-6) + \alpha^2(m^2-3m-2)}{2(m^2+m-2) + 2\alpha^2(m^2-3m+2)}. \tag{41}$$

For $\alpha^2 = 1$, there exists the following exact solution

$$f(r) = \frac{4kq^2}{m-1}r^m - 4kmMr^{m-1} + cr, \tag{42}$$

with $\Lambda = \frac{m^2-m-4}{2m(m-1)}$. Note that for $m \neq 2$ this solution exists only for $M = 0$. For $m = 0, 1$, the solution doesn't exist since Λ diverges.

For $h(r) = r^2 + \frac{2M(\alpha^2-1)}{\alpha^2}r$ and $\Lambda = \frac{-\alpha^2}{2}$, one can also find solutions like

$$f(r) = 2(1+\alpha^2)\left(\frac{\alpha^2-2}{\alpha^2-3}r^{3-2N} + M\frac{\alpha^2-2}{\alpha^2}r^{2-2N} + 2M^2\frac{1-\alpha^2}{\alpha^4}r^{1-2N} + cr^{2N}\right), \tag{43}$$

with $N = \frac{\alpha^2}{1+\alpha^2}$ and c constant.

5. SOLUTIONS WITH A GENERAL COUPLING PARAMETER AND TWO LIOUVILLE POTENTIALS

In this section, we present rotating solutions to the EMd gravity equations with infinitesimal rotation parameter and dilaton potential

$$V(\phi) = 2\Lambda_1 e^{2\beta_1\phi} + 2\Lambda_2 e^{2\beta_2\phi}, \tag{44}$$

where Λ_1 and Λ_2 are constants. This generalizes further the potential (24). If $\beta_1 = \beta_2$, then (44) reduces to (24), so we will not repeat these solutions. Requiring $\beta_1 \neq \beta_2$, from equations (8)–(11), one obtains

$$U(r) = r^{2-2N}\left(1 - 2\Lambda_1 - \frac{2M}{Nr} + \frac{\Lambda_2(1+\alpha^2)^2}{\alpha^2(1-3\alpha^2)}r^{2(2N-1)}\right), \tag{45}$$

with $N = \frac{\alpha^2}{1+\alpha^2}$ and M is the quasilocal mass. In order to fully satisfy the system of equations, the β_1 and β_2 parameters must satisfy $\beta_1 = \frac{1}{\beta_2} = -\alpha$. For the electric charge, one obtains

$$q^2 = \frac{1 + \Lambda_1(\alpha^2 - 1)}{1 + \alpha^2}, \tag{46}$$

Obviously, another solution with the same spacetime metric is generated via the discrete transformation $\beta_1 \longleftrightarrow \beta_2$ and $\Lambda_1 \longleftrightarrow \Lambda_2$.

Note that in the particular case $\Lambda_2 = 0$, this solution reduces to (37) and when $\Lambda_1 = 0$, it reduces to (25). In order to investigate the causal structure of the solution and subsequently find the horizons (similar to what was done in the previous section) we find the zeros of the function

$$F(r) = -\frac{2M}{N} + (1 - 2\Lambda_1)r + \frac{\Lambda_2(1+\alpha^2)}{\alpha^2(1-3\alpha^2)} r^{4N-1}, \tag{47}$$

We consider the cases $\alpha^2 > 1/3$ and $\alpha^2 < 1/3$, separately. For the first case, we certainly have extremum if $\Lambda_1 > 1/2(\Lambda_1 < 1/2)$ or $\Lambda_2 < 0(\Lambda_2 > 0)$. The sign of the second derivative will show whether we have local minimum or maximum. Here, for $1/3 < \alpha^2 < 1(\alpha^2 > 1)$ and $\Lambda_2 > 0(\Lambda_2 < 0)$ the function $f(r)$ would have local minimum and in opposite, for $1/3 < \alpha^2 < 1(\alpha^2 > 1)$ and $\Lambda_2 < 0(\Lambda_2 > 0)$ the function will have local maximum at

$$r_{\min(\max)} = \left(\frac{(1-2\Lambda_1)\alpha^2}{\Lambda_2(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}}. \tag{48}$$

The value of the function $F(r)$ at its extremum is

$$F(r_{\text{extr}}) = -\frac{2M}{N} + \frac{(1-2\Lambda_1)(2-3\alpha^2)}{1-3\alpha^2}\left(\frac{(1-2\Lambda_1)\alpha^2}{\Lambda_2(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} \tag{49}$$

In order to have any horizon, $F(r_{\min})(F(r_{\max}))$ should be larger(less) than or equal to zero in order to possess any local extremum and subsequently to have any horizon for the black hole. The case $F(r_{\min})(F(r_{\max})) = 0$ corresponds to an extremal black hole. The condition $F(r_{\min}) \leq 0$ gives

$$M \geq \frac{\alpha^2(2-3\alpha^2)(1-2\Lambda_1)}{2(1+\alpha^2)(1-3\alpha^2)}\left(\frac{(1-2\Lambda_1)\alpha^2}{\Lambda_2(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} \tag{50}$$

and we obtain the following inequality for the condition $F(r_{\max}) \geq 0$

$$M \leq \frac{\alpha^2(2-3\alpha^2)(1-2\Lambda_1)}{2(1+\alpha^2)(1-3\alpha^2)}\left(\frac{(1-2\Lambda_1)\alpha^2}{\Lambda_2(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}}. \tag{51}$$

For the second case where $\alpha^2 < 1/3$ the function $F(r)$ possesses local minimum for $\Lambda_2 < 0$ and local maximum for $\Lambda_2 > 0$. In this case, the function diverges both at $r = 0$ and at infinity. The local minimum (maximum) happens at (48) and the value of the function $F(r_{\min})$ is given by (49). Since in this case we have both a local minimum and maximum, condition (51), should hold for both cases.

We see that in both cases we obtain horizons for any given value of the parameter α. Here we express the radius of the extremal solution as in the preceding section

$$r_{\text{ext}} = \left(\frac{(1-2\Lambda_1)\alpha^2}{\Lambda_2(\alpha^2+1)}\right)^{\frac{1}{2}\frac{\alpha^2+1}{\alpha^2-1}} = \frac{2(1+\alpha^2)(1-3\alpha^2)}{\alpha^2(2-3\alpha^2)(1-2\Lambda_1)}M \tag{52}$$

Here, we are interested in finding the rotating version of this static solution, i.e. solving the corresponding coupled equations for two unknown functions $f(r)$ and $h(r)$. By the anzats (18) we can distinguish different solutions:

For $m = 0$, ones again we have solution of the form (19). In particular case $k = 0$, we have unusual solution where there is a change in the metric from the non-rotating form without any change in the Maxwell field also exist as before.

In the large α limit, and $q = 0$, one obtains

$$f(r) = \frac{-km\Lambda_2}{3}r^{m+3} - 2kmMr^m + cr^2. \tag{53}$$

with $m = -1, -4$ and $\Lambda_1 = \frac{m^2-m-4}{2m(m-1)}$. Note that for $m = -4$ this solution exists only for $M = 0$. For $m = -1$, the solution exists for M non zero. For *asymptotic* solutions once again we have solutions of the form

$$f(r) = r^{2N}\left(c + \frac{4kq^2}{\gamma}r^\gamma\right). \tag{54}$$

with $\gamma = m + 1 - 4N$ and c is a constant. In this case Λ is related to m and α via

$$\Lambda_1 = \frac{(m^2+m-6)+\alpha^2(m^2-3m-2)}{2(m^2+m-2)+2\alpha^2(m^2-3m+2)}. \tag{55}$$

For $\alpha^2 = 1$, there exists an exact solution in the form

$$f(r) = \frac{4kq^2}{m-1}r^m - 4kmMr^{m-1} + cr. \tag{56}$$

In this case, Λ_2 is related to m and Λ_1 via

$$\Lambda_2 = \frac{(m^2-m-4)-2\Lambda_1(m^2-m)}{2m(m-1)}. \tag{57}$$

It is notable that for $m \neq 2$ this solution exists only for $M = 0$. For $m = 0, 1$, the solution doesn't exist since Λ diverges.

6. CONCLUSION

In summary, we considered exact, electrically charged, static and spherically symmetric black hole solutions to four dimensional Einstein-Maxwell-dilaton gravity without potential or with one or two Liouville type potentials. These black holes have unusual asymptotics. They are neither asymptotically flat nor asymptotically (anti-) de Sitter.

We have added an infinitesimal rotation represented by the parameter a. In this case we need only to know a few extra components of the gauge field and the metric. These are A_ϕ, A_t and $g_{t\phi}$ which are of order a. For small angular momentum, the field equations led to the coupled differential equations satisfied by two unknown functions $f(r)$ and $h(r)$, for which we presented several classes of solutions.

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